UNSTEADY BOUNDARY LAYER WITH SELF-INDUCED PRESSURE NEAR RAPIDLY HEATED SEGMENT OF FLAT PLATE IN SUPERSONIC FLOW

A. V. Kazakov

UDC 532.526.5:533.6.011.5

The study of unsteady boundary layer flows with self-induced pressure is of great theoretical and practical interest in modern aerodynamics. The use of matched asymptotic expansions for the analysis of such flows [1-8] made it possible to establish the asymptotic nature of the flow as the characteristic Reynolds number tends to infinity, similarity laws, and also the momentum laws which agree well with experimental data at large subcritical Reynolds numbers. As in the case of purely stationary flows [9, 10], the interaction of unsteady boundary layer with inviscid external flow has a significant effect on the nature of the flow mainly in certain localized regions of the flow with longitudinal dimensions of the order of $l\,\mathrm{Re}^{-3/8}$ [1-10]. For time intervals $\Delta t \sim (l/u_\infty)\mathrm{Re}^{-1/4}$ the flow in the two regions of the flow with transverse scales of the order of $l\,\mathrm{Re}^{-3/8}$ and $l\,\mathrm{Re}^{-1/2}$, respectively, are quasisteady [1-8]. At the same time the flow in the viscous wall region whose thickness is on the order of $l\,\mathrm{Re}^{-5/8}$ happens to be appreciably unsteady and is described by unsteady, incompressible boundary layer equations. The pressure gradient in these equations is not specified as in Prandtl's boundary layer theory but determined during the process of solution of the problem from the condition of viscous sublayer interaction with external supersonic flow. The body surface temperature was assumed constant in [1-10] along the entire interaction region. Thanks to this, the density and dynamic viscosity of the fluid in the entire viscous region with thickness of the order of $l\,\mathrm{Re}^{-5/8}$ are constant to the first approximation and their values coincided with respective values in undisturbed boundary layer on the body surface. In this case the solution of the equations of motion in the viscous region can be found independently of the energy equation whose solution is determined later from the computed velocity field [1-8]. The present work investigates the interaction of laminar boundary layer and supersonic flow arising from variation in temperature of the small surface region of the body by an amount equal to the order of the surface temperature itself over a characteristic time $\Delta t \sim (l/u_\infty)\mathrm{Re}^{-1/4}$.

Consider an unsteady flow caused by heating a segment of a flat plate in supersonic flow as the characteristic Reynolds number $\mathrm{Re} = \rho_\infty u_\infty l/\mu_\infty = \varepsilon^{-2}$ approaches infinity. Here ρ_∞, u_∞, μ_∞ are the density, velocity, and absolute viscosity of the free stream, l is the distance from the flat plate leading edge to that segment of the surface which is subjected to rapid heating by any internal or external energy source. For convenience all linear dimensions are referred to l, velocity components to u_∞, density to ρ_∞, time to l/u_∞, pressure to $\rho_\infty u_\infty^2$, enthalpy to u_∞^2, and dynamic viscosity to μ_∞, and in what follows only dimensionless quantities will be used. Assume that the characteristic length of the heated segment is on the order $\varepsilon^{3/4}$, and the surface enthalpy (or temperature) varies by a value of the order of unity over a characteristic time $\Delta t \sim \varepsilon^{1/2}$. In this case it is possible to distinguish three different regions of the flow with equal length $\sim\varepsilon^{3/4}$ in the neighborhood of the heated segment: inviscid region of the supersonic flow (region 1), whose streamwise and transverse dimensions are of the same order $\Delta x \sim y \sim \varepsilon^{3/4}$; the inviscid vortex flow whose transverse scale is of the order of the undisturbed boundary layer $y \sim \varepsilon$ (region 2); viscous wall layer (region 3) with a thickness of the order $\varepsilon^{5/4}$ in which the velocity and enthalpy fluctuations are of the same order as the velocity and enthalpy at the surface of the body in the undisturbed boundary layer upstream of the interaction region.

Asymptotic expansions for coordinates and flow parameters in the region 1 can be represented in the form

$$x - 1 = \varepsilon^{3/4}x_1,\ y = \varepsilon^{5/4}y_1,\ t = \varepsilon^{1/2}t_1, \tag{1}$$
$$u = 1 + \varepsilon^{1/2}u_{11}(t_1, x_1, y_1) + \ldots,\ p = 1/\gamma M_\infty^2 + \varepsilon^{1/2}p_{11}(t_1, x_1, y_1).$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 38-43, January-February, 1983. Original article submitted December 7, 1981.

$$v = \varepsilon^{1/2} v_{11}(t_1, x_1, y_1) + \ldots, \; h = 1/[(\gamma - 1)M_\infty^2] + \varepsilon^{1/2} h_{11}(t_1, x_1, y_1), \quad \rho = 1 + \varepsilon^{1/2}\rho_{11}(t_1, x_1, y_1) + \ldots \tag{1}$$

Here M_∞ is the free stream Mach number. The substitution of asymptotic expansions (1) in Navier–Stokes equations and the limiting case $\varepsilon \to 0$ show that, as in [1-10], the flow in the region 1 is a weakly disturbed supersonic flow and is described by linear supersonic flow theory. The solution to the wave equation is determined from D'Alembart's equation which makes it possible to obtain a relation between pressure disturbance p_{11} and vertical velocity v_{11} at $y_1 = 0$:

$$p_{11}(t_1, x_1, 0) = \frac{1}{\sqrt{M_\infty^2 - 1}} v_{11}(t_1, x_1, 0). \tag{2}$$

Note that the flow parameters at $y_1 = 0$ are determined by matching asymptotic expansions for regions 1 and 2.

The flow in the region 2 consisting of the bulk of the flow in the undisturbed boundary layer, in the first approximation, as shown in [1-8], is locally inviscid and does not affect the pressure distribution in the interaction region. Asymptotic expansions and equations describing the flow in this region as well as their solutions are obtained in the present case in a manner similar to [8]. Note, however, that time derivatives of flow parameters are absent in equations describing the flow in regions 1, 2 and the solutions depend on the variable t as a parameter [6-8].

Asymptotic expansions for flow parameters in the viscous wall layer close to the surface (region 3) have the following form:

$$\begin{gathered} x - 1 = \varepsilon^{3/4} x_3, \; y = \varepsilon^{5/4} y_3, \; t = \varepsilon^{1/2} t_3, \\ u = \varepsilon^{1/4} u_{31}(t_3, x_3, y_3) + \ldots, \; p = 1/\gamma M_\infty^2 + \varepsilon^{1/2} p_{31}(t_3, x_3, y_3), \\ v = \varepsilon^{3/4} v_{31}(t_3, x_3, y_3) + \ldots, \; h = h_{30}(t_3, x_3, y_3) + \ldots, \\ \rho = \rho_{30}(t_3, x_3, y_3) + \ldots, \; \mu = \mu_{30}(t_3, x_3, y_3) + \ldots \end{gathered} \tag{3}$$

The substitution of expansion (3) in Navier–Stokes equations with the limiting case $\varepsilon \to 0$ and also matching the asymptotic expansions in region 1-3 in order to determine the required boundary conditions [8] make it possible to obtain the following boundary-value problem for the region 3:

$$\begin{gathered} \frac{\partial \rho_{30}}{\partial t_3} + \frac{\partial \rho_{30} u_{31}}{\partial x_3} + \frac{\partial \rho_{30} v_{31}}{\partial y_3} = 0, \; \frac{\partial p_{31}}{\partial y_3} = 0, \\ \rho_{30}\frac{\partial u_{31}}{\partial t_3} + \rho_{30} u_{31}\frac{\partial u_{31}}{\partial x_3} + \rho_{30} v_{31}\frac{\partial u_{31}}{\partial y_3} = -\frac{\partial p_{31}}{\partial x_3} + \frac{\partial}{\partial y_3}\left(\mu_{30}\frac{\partial u_{31}}{\partial y_3}\right), \\ \rho_{30}\frac{\partial h_{30}}{\partial t_3} + \rho_{30} u_{31}\frac{\partial h_{30}}{\partial x_3} + \rho_{30} v_{31}\frac{\partial h_{30}}{\partial y_3} = \frac{1}{\sigma}\frac{\partial}{\partial y_3}\left(\mu_{30}\frac{\partial h_{30}}{\partial y_3}\right), \\ \rho_{30} h_{30} = 1/[(\gamma - 1)M_\infty^2], \; \mu_{30} = [(\gamma - 1)M_\infty^2]^\omega h_{30}^\omega, \\ p_{31} = \frac{1}{\sqrt{M_\infty^2 - 1}} \frac{d}{dx_3}\left[\lim\left(y_3 - \frac{u_{31}}{a_0}\right)\right], \\ u_{31}(0, x_3, y_3) = a_0 y_3, \; h_{30}(0, x_3, y_3) = h_{20}(0) = h_{00}, \; p_{31}(0, x_3) = 0, \end{gathered} \tag{4}$$

$$\frac{\partial u_{31}}{\partial y_3} \to a_0, \; h_{30} \to h_{00} \text{ as } y_3 \to +\infty \text{ or } x_3 \to -\infty, \quad u_{31}(t_3, x_3, 0) = v_{31}(t_3, x_3, 0) = 0, \; h_{30}(t_3, x_3, 0) = h_W(t_3, x_3).$$

Here and in what follows the index W indicates flow parameters at the wall; ω is the index in the relation for the variation of viscosity with temperature; h_{00} is the stagnation enthalpy at the plate surface in undisturbed boundary layer with $y_2 = 0$ ($h_{20}(0) = h_{00}$). Enthalpy h_{30} is eliminated from the system of equations (4) using the equation of state and the following new variables are introduced:

$$\begin{gathered} x_3 = \frac{\rho_{00}^{\frac{\omega-2}{4}}}{\beta^{3/4} a_0^{5/4}} X, \; y_3 = \frac{1}{\beta^{1/4} a_0^{3/4} \rho_{00}^{\frac{\omega+2}{4}}} Y, \\ t_3 = \frac{\rho_{00}^{\omega/2}}{\beta^{1/2} a_0^{3/2}} T, \; u_{31} = \frac{a_0^{1/4}}{\beta^{1/4} \rho_{00}^{\frac{\omega+2}{4}}} U, \\ v_{31} = \frac{a_0^{3/4} \beta^{1/4}}{\rho_{00}^{\frac{3\omega+2}{4}}} V, \; p_{31} = \frac{a_0^{1/2}}{\beta^{1/2} \rho_{00}^{\omega/2}} P, \quad \rho_{30} = \rho_{00} R, \quad \mu_{30} = \rho_{00}^{-\omega} M, \end{gathered} \tag{5}$$

where ρ_{00} is the density in the undisturbed boundary layer for the surface temperature upstream of the interaction region with $y_2 = 0$; $\beta = (M_\infty^2 - 1)^{1/2}$. In the new variables (5) equations and boundary conditions (4) for the viscous wall layer of the interaction region take the form

$$\frac{\partial R}{\partial T} + \frac{\partial RU}{\partial X} + \frac{\partial RV}{\partial Y} = 0, \tag{6}$$

$$R\frac{\partial U}{\partial T} + RU\frac{\partial U}{\partial X} + RV\frac{\partial U}{\partial Y} = -\frac{dP}{dX} + \frac{\partial}{\partial Y}\left(M\frac{\partial U}{\partial Y}\right),$$

$$\frac{\partial R}{\partial T} + U\frac{\partial R}{\partial X} + V\frac{\partial R}{\partial Y} = \frac{R}{\sigma}\frac{\partial}{\partial Y}\left(\frac{M}{R^2}\frac{\partial R}{\partial Y}\right),$$

$$M = R^{-\omega}, \quad P = \frac{d}{dX}\left[\lim_{Y\to\infty}(Y - U)\right],$$

$$U(0, X, Y) = Y, \quad R(0, X, Y) = 1, \quad P(0, X) = 0,$$

$$\frac{\partial U}{\partial Y} \to 1, \quad R \to 1 \text{ as } Y \to +\infty \text{ or } X \to -\infty,$$

$$U(T, X, 0) = V(T, X, 0) = 0, \quad R(T, X, 0) = R_W(T, X).$$

A solution has been obtained in the present paper for the boundary-value problem (6) for the case of a small increase in surface temperature, or, in other words, a small variation in fluid density at the surface: $R_W = 1 + \delta R_{1W}$ $(0 < \delta \ll 1)$. The boundary-value problem (6) can be linearized in this case using the small parameter δ and seeking a solution in the form $R = 1 + \delta R_1$, $U = Y + \delta U_1$, $V = \delta V_1$, $P = \delta P_1$.

The following system of equations and boundary conditions are obtained in this case for the fluctuating flow parameters:

$$\frac{\partial R_1}{\partial T} + Y\frac{\partial R_1}{\partial X} + \frac{\partial U_1}{\partial X} + \frac{\partial V_1}{\partial Y} = 0, \tag{7}$$

$$\frac{\partial U_1}{\partial T} + Y\frac{\partial U_1}{\partial X} + V_1 = -\frac{dP_1}{dX} + \frac{\partial^2 U_1}{\partial Y^2} - \omega\frac{\partial R_1}{\partial Y},$$

$$\frac{\partial R_1}{\partial T} + Y\frac{\partial R_1}{\partial X} = \frac{1}{\sigma}\frac{\partial^2 R_1}{\partial Y^2}, \quad P_1 = -\frac{dA_1}{dX}, \quad A_1 = \lim_{Y\to\infty}(U_1),$$

$$U_1(0, X, Y) = 0, \quad R_1(0, X, Y) = 0, \quad P_1(0, X) = 0,$$

$$\frac{\partial U_1}{\partial Y} \to 0, \quad R_1 \to 0 \text{ as } Y \to +\infty \text{ or } X \to -\infty,$$

$$U_1(T, X, 0) = V_1(T, X, 0) = 0, \quad R_1(T, X, 0) = R_{1W}(T, X).$$

Finite-difference method with implicit scheme with respect to time was used to solve system (7). The system of differential equations (7) was replaced by the corresponding difference equations of first-order accuracy with respect to the variables T and X, and second-order accuracy with respect to the variable Y. The solution of the system of difference equations was found using relaxation method with iterations at each time layer.

The density field $R_1(T_{i+1}, X, Y)$ was found at the time layer T_{i+1} and then the pressure distribution was specified from which the velocity field, and, in particular $A_1(T_{i+1}, X)$ was determined. Then the variation in the displacement thickness A_1 thus obtained was subjected to relaxation and a new pressure distribution was obtained. The procedure is continued till the difference in the fluctuations in viscous stress $\partial U_1/\partial Y$ and displacement thickness at two consecutive iterations become less than a certain small specified value. In order to start the iteration process at the T_{i+1} layer the pressure distribution from the pressure layer at time T_i was used. At the initial time $T = 0$ the fluctuation in flow parameters was assumed zero. As an example the results of the computation of unsteady flow in viscous sublayer with Prandtl number $\sigma = 1$ are given where the variation in density at the surface with time T and streamwise coordinate X are specified as follows:

$$R_{1W}(T, X) = \begin{cases} -\sin\left(\frac{\pi}{2}T\right)\exp(-BX^2), & 0 \leqslant T \leqslant 1, \\ -\exp(-BX^2), & T > 1. \end{cases} \tag{8}$$

At time $T = 0$ the density fluctuation on the surface with $Y = 0$ equals zero and the flow in the interaction region remains undisturbed. At the following instants of time as $T > 0$ the fluid density at the surface $Y = 0$ begins to decrease $(0 < \delta \ll 1,\ R_{1W} < 0)$ which corresponds to an increase in surface temperature in this region. In this case the viscous wall layer begins to heat up, leading to a change in shear stress at the surface $\partial U_1/\partial Y = (c_f - c_{f0})/\delta c_{f0} + \omega R_{1W}$ with c_{f0} being the nondimensional skin-friction coefficient in the undisturbed boundary layer upstream of the interaction region and the parameters B and ω in the boundary-value problem (7) were assumed equal to one $(B = \omega = 1)$. Curves 1-3 represent the distribution of the quantity $\partial U_1/\partial Y$ on

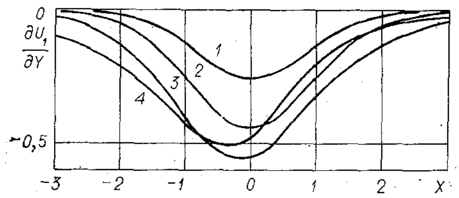

Fig. 1

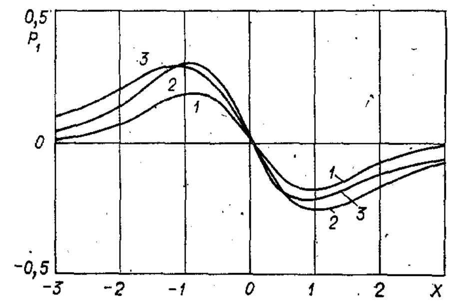

Fig. 2

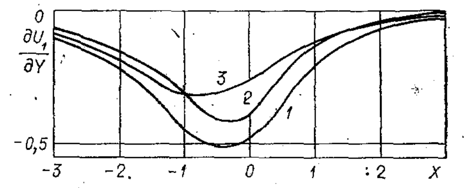

Fig. 3

the flat plate surface in the interaction region at moments $T = 0.25, 0.5, 1.0$, respectively. The curve 4 represents stationary distribution of the disturbed shear stress $\partial U_1/\partial Y$ obtained after establishing the flow in the viscous sublayer of the interaction region ($T \gtrsim 10$). Shear stress, as indicated by computations, most appreciably decreases in the region of maximum variation in density $R_{1W}(T, X)$ or in the region of maximum increase in surface temperature ($X = 0$). At time $T = 1.0$ the density at the surface at $X = 0$ reaches a minimum value at the origin ($X = 0$) (surface temperature in this case has a maximum at $X = 0$). At this moment $T = 1$ shear stress attains its minimum value $\partial U_1/\partial Y = -0.56$ at $X = -0.2$. Then while establishing the flow at $T > 1.0$ the adverse pressure gradient in the interaction region begins to decrease, thanks to which even the maximum value of the fluctuation in shear stress at the surface decreases. The distribution of pressure fluctuation $P_1(T, X)$ in the interaction region is shown in Fig. 2, where the curves 1-3 correspond to moments $T = 0.5$, 1.0, 10.5. Curve 3 represents the stationary distributions of the pressure fluctuation obtained after establishing the flow in the viscous sublayer. These computational results show that the local surface heating can lead to a reduction in shear stress in a certain region of the surface and even to a local flow separation and a significant redistribution of pressure along the surface of the body, and, consequently, to a change in its momentum characteristics. Numerical results showed that for low values of ω fluctuations in shear stress and pressure become less than in the case $\omega = 1$ in the interaction region near the heated segment of the surface. The stationary distributions of fluctuating shear stress $\partial U_1/\partial Y$ along the surface obtained by solving the boundary-value problem (7), (8) for the cases $\omega = 1$, $B = 1$ (curve 1) and $\omega = 0.5$, $B = 1.0$ (curve 2) are shown in Fig. 3. Curve 3 is for the stationary distribution with respect to the shear stress for the case $\omega = 1.0$, $B = 2.0$ corresponding to the interaction of nonstationary boundary layer and supersonic flow near the surface region where the heating is relatively less. The variation in surface shear stress also decreases in this case when compared to the cases $\omega = 1$, $B = 1$.

In conclusion, we note that similarity parameters obtained by the introduction of new variables (5) make it possible to analyze the effect of various flow parameters on the nonstationary flow with interaction near the heated segments of the body surface.

LITERATURE CITED

1. P. G. Daniels, "The flow about the trailing edge of a supersonic oscillating aerofoil," J. Fluid Mech., 72, No. 3 (1975).
2. S. N. Brown and P. G. Daniels, "On viscous flow about the trailing edge of a rapidly oscillating plate," J. Fluid Mech., 67, No. 4 (1975).
3. O. S. Ryzhov and E. D. Terent'ev, "Nonstationary boundary layer with self-induced pressure," Prikl. Mat. Mekh., 41, No. 6 (1977).
4. V. I. Zhuk and O. S. Ryzhov, "Solutions of dispersion equation using the theory of free boundary-layer interaction," Dokl. Akad. Nauk SSSR, 247, No. 5 (1979).
5. E. D. Terent'ev, "Nonstationary boundary layer with self-induced pressure near oscillating wall in supersonic flow," Dokl. Akad. Nauk SSSR, 240, No. 5 (1978).
6. A. I. Ruban, "Numerical solution of local asymptotic problem of unsteady separation of laminar boundary layer in supersonic flow," Zh. Vychisl. Mat. Mat. Fiz., No. 5 (1978).
7. A. V. Kazakov, "Flat plate with flap in unsteady supersonic viscous flow," Uch. Zap. Central Hydrodynamic Institute, 11, No. 4 (1980).
8. O. S. Ryzhov, "Unsteady three-dimensional boundary layer, freely interacting with external flow," Prikl. Mat. Mekh., 44, No. 6 (1980).
9. K. Stewartson and P. G. Williams, "Self-induced separation," Proc. R. Soc. London, Ser. A, 312 (1969).
10. V. Ya. Neiland, "Theory of laminar boundary-layer separation in supersonic flow," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1969).

TEMPERATURE DISTRIBUTION OVER THE SURFACES OF SPHERICAL SHELLS IN A PURGED DENSE LAYER WITH INTERNAL HEAT GENERATION

L. K. Vukovich, V. I. Lelekov,
A. V. Nikolaev, S. S. Timar',
and N. V. Tkach

UDC 536.244:621.039

Instruments are being developed at present in which a spherical filling is enclosed between perforated walls and its thickness amounts to three-six particle diameters. The specific conditions for the entry of gas into the filling (through holes in the perforated wall) and its relatively small thickness should have an effect on the nature of the gas motion in the filling, and consequently on heat exchange with spheres placed in various arrays. The distribution of the local characteristics of heat or mass exchange through the surface of spheres in the packings has been investigated in a series of experimental researches. The results obtained are presented in the form of a distribution of the local coefficients of heat exchange over the surface. However, a number of practical problems require knowledge of the local surface temperatures (for example, for the calculation of the thermal stresses in the casings enveloping a heat-generating sphere), which it is impossible to determine from the existing local heat transfer coefficients determined by detectors of the local thermal and mass fluxes, in connection with the interrelationship between the internal and external heat exchange problems [1]. An approximate computational dependence has been proposed in [2] for the determination of the maximum temperature nonuniformity in the casing enveloping a heat-generating core. This dependence has been derived for a single type of packing of the spheres. The absence in it of the heat-generation power remains incomprehensible. An expression for the relative maximum increase of the temperature differential in the casing caused by the different intensity of heat exchange at various points of its surface has been obtained in [3] by an alternate numerical solution of the time-independent thermal conductivity equation for a spherical heat-generating element under boundary conditions of the third kind (determined experimentally) in the range of variation 0.4-2.85 of the ratios of the thermal conductivity of the shell material and the coolant. However, this dependence has been derived for a specific packing of the spheres with a ratio of the channel and sphere diameters of less

Odessa. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 43-48, January-February, 1983. Original article submitted October 20, 1981.